

Determination of the $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ resonance parameters

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Abstract

R measurement data taken with the BESII detector at center-of-mass energies between 3.7 and 5.0 GeV is fitted to determine resonance parameters (mass, total width, electron width) of the high mass charmonium states, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$. Various effects, including the relative phases between the resonances, interferences, the energy-dependence of the full widths, and the initial state radiative correction, are examined. The results are compared to previous studies.

1 Introduction

The total cross section for hadron production in e^+e^- annihilation is usually parameterized in terms of the ratio R , which is defined as $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, where the denominator is the lowest-order QED cross section, $\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \sigma_{\mu\mu}^0 = 4\pi\alpha^2/3s$. The measured R values are consistent with the three-color quark model predictions. At the open flavor thresholds where resonance structures show up, measurements of R value are used to determine resonance parameters. For the high mass charmonium resonances, the $\psi(3770)$ was measured by MARK-I [1], DELCO [2], MARK-II [3] and BES [4][5]; the $\psi(4040)$ and $\psi(4160)$ were measured by DASP [6]; and the $\psi(4415)$ was measured by DASP [6] and MARK-I [7]. There were also some other measurements of R values as reported in Refs. [8, 9, 10], but no attempt was made to determine resonance parameters. The resonance parameters in the Particle Data Group (PDG)'s compilation remained unchanged for more than 20 years up to the 2004 edition [11]. The resonance parameters for the three high mass resonances were updated by PDG2006 [12], based on K. Seth's evaluation [13] using combined BESII [14] and Crystal Ball [10] data.

The most recent and precise R measurements between 2-5 GeV were made by BESII [14]. Experimentally, R for both the continuum and the wide resonance region is given by

$$R_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had} (1 + \delta_{obs})}, \quad (1)$$

where N_{had}^{obs} is the number of observed hadronic events, N_{bg} is the number of the residual background events, L is the integrated luminosity, $(1 + \delta_{obs})$ is the effective correction factor of the initial state radiation (ISR) [10][15], ϵ_{had} is the detection efficiency for hadronic events determined by the Monte Carlo simulation without bremsstrahlung being simulated, and ϵ_{trg} is the trigger efficiency. The determination of R values and resonance parameters are intertwined; the factor $(1 + \delta_{obs})$ in Eq. (1) contains contributions from the resonances and depends on the resonance parameters. Therefore, the procedure to calculate $(1 + \delta_{obs})$ requires a number of iterations before stable results can be obtained.

In this work, we perform a global fit over the entire center-of-mass energy region from 3.7 to 5.0 GeV covering the four resonances, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$, and include interference effects among the resonances. We also adopt energy-dependent full widths, and introduce relative phases between the resonances. Finally, the new R values due to the updated resonant parameters are compared to the published ones [14].

2 Fitting of the resonant parameters

The phenomenological models and formulas used in our fitting are briefly described below.

2.1 Breit-Wigner form

The relativistic Breit-Wigner amplitude for $e^+e^- \rightarrow \text{resonance} \rightarrow \text{final state } f$ is

$$\mathcal{T}_r^f(W) = \frac{M_r \sqrt{\Gamma_r^{ee} \Gamma_r^f}}{W^2 - M_r^2 + i M_r \Gamma_r} e^{i\delta_r}, \quad (2)$$

where $W \equiv E_{cm} \equiv \sqrt{s}$ is the center-of-mass energy, the index r represents the resonance to be studied, M_r is the nominal mass, Γ_r is the full width, Γ_r^{ee} is the electron width, Γ_r^f is the hadronic width for the decaying channel f , and δ_r is the phase.

The high mass charmonia decay into several two-body final states f . According to the Eichten model [16] and experimental data [17], the decay channels (including their conjugate states) are:

$$\begin{aligned} \psi(3770) &\Rightarrow D\bar{D}; \\ \psi(4040) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s; \\ \psi(4140) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s, D_s\bar{D}_s^*; \\ \psi(4415) &\Rightarrow D\bar{D}, D^*\bar{D}^*, D\bar{D}^*, D_s\bar{D}_s, D_s\bar{D}_s^*, D_s^*\bar{D}_s, D\bar{D}_1, D\bar{D}_2^*. \end{aligned}$$

The total squared inclusive amplitude of the resonances is the incoherent sum over all different decay channels f ,

$$|\mathcal{T}_{res}|^2 = \sum_f \left| \sum_r \mathcal{T}_r^f(W) \right|^2. \quad (3)$$

The resonant cross section expressed as the R value is then given by

$$R_{res} = \frac{\sigma_{res}}{\sigma_{\mu\mu}^0} = \frac{12\pi}{s} |\mathcal{T}_{res}|^2. \quad (4)$$

2.2 Energy-dependence of the full width

The full width of a broad resonance depends on the energy. A phenomenological model derived from quantum mechanics is used to describe the behavior of $\Gamma_r^f(W)$, which depends on the momentum and the orbital angular momentum L of the decaying final state [18],

$$\Gamma_r^f(W) = \hat{\Gamma}_r \frac{2M_r}{M_r + W} \sum_L \frac{Z_f^{2L+1}}{B_L}, \quad (5)$$

where, $\hat{\Gamma}_r$ is a parameter to be determined by fitting experimental data, $Z_f \equiv \rho P_f$, ρ is the radius of the interaction with the order of a few fermis (the value is insensitive to the physical results), P_f is the decay momentum, and $2M_r/(M_r + W)$ is a relativistic correction factor. The energy-dependent partial wave functions B_L are given in Ref. [18]:

$$\begin{aligned} B_0 &= 1, \quad B_1 = 1 + Z^2, \quad B_2 = 9 + 3Z^2 + Z^4, \\ B_3 &= 225 + 45Z^2 + 6Z^4 + Z^6. \end{aligned} \quad (6)$$

When the resonance decays to several hadronic channels, the total hadronic width is the sum of all its partial widths,

$$\Gamma_r^{had}(W) = \sum_f \Gamma_r^f(W). \quad (7)$$

The total width of the resonance r is expressed as

$$\Gamma_r(W) = 3\Gamma_r^{ee} + \Gamma_r^{had}(W), \quad (8)$$

where the universality of the leptons is used, i.e. $\Gamma_r^{ee} = \Gamma_r^{\mu\mu} = \Gamma_r^{\tau\tau}$, and Γ_r^{ee} is the experimental electron width, which includes the contribution from the vacuum polarization effect.

2.3 Continuum background

The contribution of continuum background originating from initial light quark pairs $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$, which is written as $R_{QCD}^{(uds)}$, may be predicted by pQCD above 2 GeV[12]. Being close to the production threshold, the continuum open- $c\bar{c}$ background can only be described by phenomenological models or empirical expressions. Since there are many possible channels above the open-charm threshold, and their cross sections are expected to vary smoothly, we parametrize, for simplicity, the continuum charm background with a second order polynomial,

$$R_{con}^{(c)} = C_0 + C_1(W - 2M_{D^\pm}) + C_2(W - 2M_{D^\pm})^2, \quad (9)$$

where C_0 , C_1 and C_2 are free parameters, and M_{D^\pm} is the mass of the lightest meson D^\pm .

2.4 Fitting scheme

We fit the experimental data with MINUIT [19] using a least squares method, with χ^2 defined as [20]

$$\chi^2 = \sum_i \frac{[f_c \tilde{R}_{exp}(W_i) - \tilde{R}_{the}(W_i)]^2}{[f_c \Delta \tilde{R}_{exp}^{(i)}]^2} + \frac{(f_c - 1)^2}{\sigma_c^2}, \quad (10)$$

where W_i stands for the energy of the measured point. The experimental quantity

$$\tilde{R}_{exp} = \frac{N_{had}^{obs} - N_{bg}}{\sigma_{\mu\mu}^0 L \epsilon_{trg} \epsilon_{had}}, \quad (11)$$

and the corresponding theoretical quantity

$$\tilde{R}_{the} = (1 + \delta_{obs}) R_{the}, \quad (12)$$

where,

$$R_{the} = R_{con} + R_{res}, \quad (13)$$

and

$$R_{con} = R_{(QCD)}^{(uds)} + R_{con}^{(c)}. \quad (14)$$

$\Delta \tilde{R}_{exp}^{(i)}$ in Eq. (10) is the combined statistical and uncommon systematic errors of $\tilde{R}_{exp}(W_i)$; the error common to all the points σ_c ($\sim 3.3\%$) is not included. In each iteration, the resonant parameters in the calculation of $(1 + \delta_{obs})$ and R_{the} are updated to new values. f_c is a scale factor which reflects the influence of the common error on the fitting. $R_{(QCD)}^{(uds)}$ is derived from pQCD [21].

The free parameters in the fit are M_r , Γ_r^{ee} , δ_r in Eq. (2), $\hat{\Gamma}_r$ in Eq.(5), and C_0 , C_1 , C_3 in Eq. (9). Only relative values of the phases can be extracted, so for simplicity, the phase of $\psi(3770)$ is set to zero.

3 Results and discussion

The resonant parameters of the high mass charmonia determined in this work, together with those in PDG2004, PDG2006 and the results given in Ref. [13] are listed in Table 1. The updated R values between 3.7 and 5.0 GeV and the fit curves are illustrated in Fig. 1. The fit yields $\chi^2/d.o.f = 1.06$, which indicates a reasonable fit.

In order to understand the uncertainty of the model-dependence, alternative choices and combinations of Breit-Wigner forms, energy dependence of the full width predicted by quantum mechanics model [18] or the effective interaction theory [22], and continuum charm background described by a second order polynomial or the phenomenological form used by DASP [6] are used. We find that the results are sensitive to the form of the energy dependent total width, but not sensitive to the form of background. The DASP background function has six continuum production channels, while the effective interaction theory predicts different energy-dependence of the hadronic width for different decay channels. However in both cases the best fits give

unreasonable results. This may be understood as the inclusive data can not supply enough information to determine the relative strength of different decay channels. To understand the detailed structures and components of the high mass charmonia states, it is necessary to collect data at each peak with sufficiently high statistics, and to develop more reliable physical models. This is one of the physics tasks of a tau charm factory, and might be further studied with BESIII that is under construction.

It is worth noting that the change of the resonance parameters affects the effective initial state radiative correction factors, and thus affects the R values. Fig. 2 shows comparison between the R values published in [14] and the updated values in this work; the difference varies with the resonant structure. In general the relative difference is within 3%, and for a few energy points the maximum difference is about 6%. Our results are in agreement with the previous experiments in most cases, but large differences are observed in some of the parameters, such as the mass of the $\psi(4160)$. This is mainly due to the reconsideration of the radiative correction factors, and the inclusion of interferences between the resonances.

Table 1: The resonance parameters of the high mass charmonia in this work together with the values in PDG2004 [11], PDG2006 [12] and K. Seth's evaluations [13] based on Crystal Ball and BES data. The total width $\Gamma_{tot} \equiv \Gamma_r(M)$ in Eq.(8).

		$\psi(3770)$	$\psi(4040)$	$\psi(4160)$	$\psi(4415)$
M (MeV/ c^2)	PDG2004	3769.9 \pm 2.5	4040 \pm 10	4159 \pm 20	4415 \pm 6
	PDG2006	3771.1 \pm 2.4	4039 \pm 1.0	4153 \pm 3	4421 \pm 4
	CB (Seth)	-	4037 \pm 2	4151 \pm 4	4425 \pm 6
	BES (Seth)	-	4040 \pm 1	4155 \pm 5	4455 \pm 6
	BES (this work)	3771.4 \pm 1.8	4038.5 \pm 4.6	4191.6 \pm 6.0	4415.2 \pm 7.5
Γ_{tot} (MeV)	PDG2004	23.6 \pm 2.7	52 \pm 10	78 \pm 20	43 \pm 15
	PDG2006	23.0 \pm 2.7	80 \pm 10	103 \pm 8	62 \pm 20
	CB (Seth)	-	85 \pm 10	107 \pm 10	119 \pm 16
	BES (Seth)	-	89 \pm 6	107 \pm 16	118 \pm 35
	BES (this work)	25.4 \pm 6.5	81.2 \pm 14.4	72.7 \pm 15.1	73.3 \pm 21.2
Γ_{ee} (keV)	PDG2004	0.26 \pm 0.04	0.75 \pm 0.15	0.77 \pm 0.23	0.47 \pm 0.10
	PDG2006	0.24 \pm 0.03	0.86 \pm 0.08	0.83 \pm 0.07	0.58 \pm 0.07
	CB (Seth)	-	0.88 \pm 0.11	0.83 \pm 0.08	0.72 \pm 0.11
	BES (Seth)	-	0.91 \pm 0.13	0.84 \pm 0.13	0.64 \pm 0.23
	BES (this work)	0.18 \pm 0.04	0.81 \pm 0.20	0.50 \pm 0.27	0.37 \pm 0.14
δ (degree)	BES (this work)	0	133 \pm 68	301 \pm 61	246 \pm 86

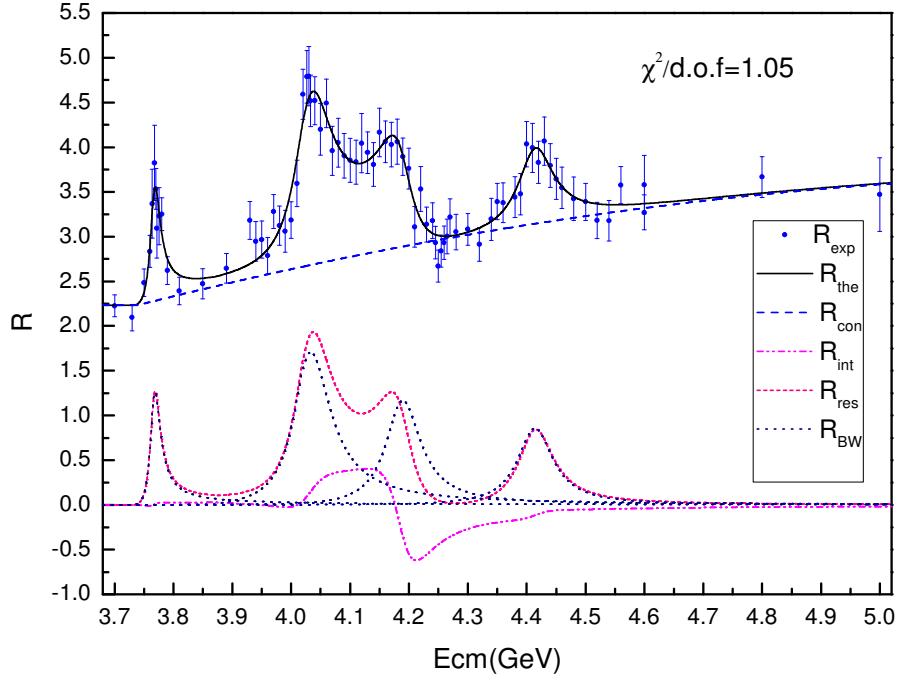


Figure 1: The fit to the R values for the high mass charmonia structure. The dots with error bars are the updated R values. The solid curve shows the best fit, and the other curves show the contributions from each resonance R_{BW} , the interference R_{int} , the summation of the four resonances $R_{res} = R_{BW} + R_{int}$, and the continuum background R_{con} respectively.

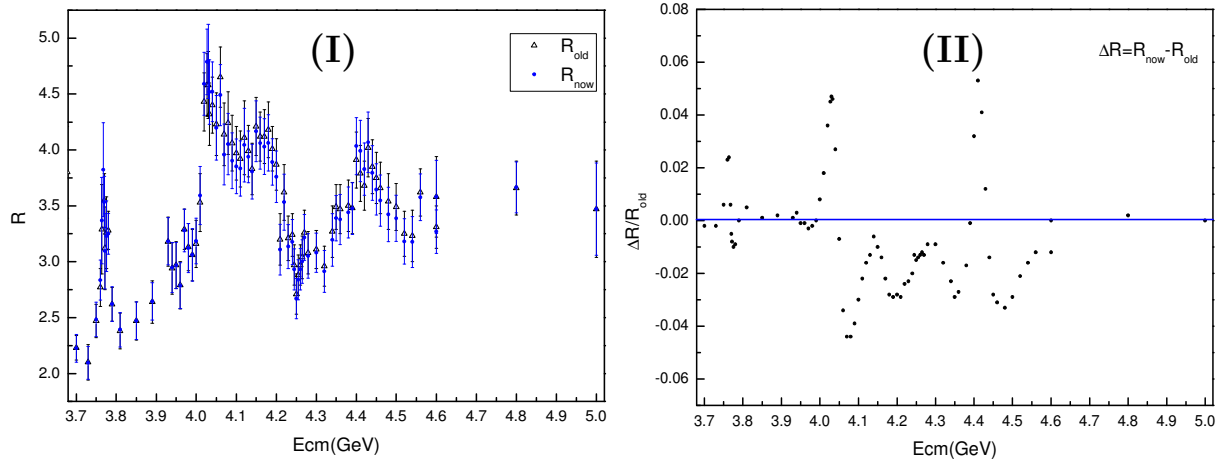


Figure 2: (I) The comparison of R values between the values published in Ref. [14] (triangles: R_{old}) and the updated values in this work (points: R_{now}). (II) The relative differences between the two sets of R values.

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